The applicability of the neoclassical microeconomic framework implied by the preference axioms

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Abstract

This thesis intends to examine how three preference axioms: transitivity, completeness and continuity, influence to the applicability of the neoclassical microeconomic framework. By postulating certain axioms on preferences of an economic agent, the theory takes an axiomatic approach to model economic behavior. Circumstances in which these preference axioms are violated express limitations of the framework. In order to be mindful of shortcomings of the framework and to get better insight into assumptions of the theory, one should be acquainted with this topic.

The thesis follows a biphasic approach. First, the critique against the preference axioms are gathered from studies in the field, then limitations of the framework are outlined based on evaluations of the findings. No attention is paid to the normative justification of the axioms.

As result of the thesis, it is posed a set of restrictions that afflicts the neoclassical microeconomic framework. All of arguments against the preference axioms are not equally problematic – this paper questions the cause of observed intransitivity: “rational intransitivity due to the high cost holding a preference ordering”, first proposed by Edwards (1954) but also appears in Weinstein (1968). Furthermore, it turns out that the axioms cannot be always treated independently.

Contents

Introduction

The neoclassical microeconomics rests on a set of assumptions, which intend to reflect how preferences of an economic agent should be modeled. There are at least two substantial reasons why to study these preference axioms. First, to get insight what economists mean when they are talking about rationality? This issue, rationality in economics, is closely related to the preference axioms. Some authors even equate the notion of the rational agent with an economic agent who has a rational preference relation[1] over possible alternatives (e.g. Peters, 2005; Autor, 2010). This is not the whole truth either. Namely, so many things can be seen as rational such as behavior, expectations or beliefs – not only preferences. Second, the appropriateness of the preference axioms crucially determines how plausible the economic theory is. Therefore arguments against the axioms should be taken seriously, although all arguments are not equally problematic. There is a lot of empirical evidence that the preference axioms postulated in the microeconomics are descriptively invalid[2]. Because of this, focus should be on how normatively adequate the axioms are. What kind of economic agent are we proposing by postulating these axioms and how does this correspond to our purposes? Do axioms restrict the applicability of our framework? The latter question is interesting for a user of the framework. It is clear that he should be mindful of limitations of the framework in question. We cannot demand that assumptions of the theory are not needed to be valid in all circumstances, rather these should be valid in circumstances relevant for the theory.

The issue comprises literature from several academic disciplines. Economists such as Frisch, Morgenstern and Samuelson likewise mathematicians such as von Neumann and Debreu have been in key roles in the axiomatization of consumer theory. Authors from other disciplines than economics have typically posed arguments against the preference axioms. Especially, the transitivity assumption has been getting a lot of attention by psychologists and philosophers. Tversky and Kahneman have successfully criticized the assumptions of rationality prevailing in neoclassical economics. On the other hand, economists have commonly provided normative justifications for the preference axioms.[3]

[1] Also known as *a complete and transitive preference relation* – two main properties of a preference relation that will be studied in this thesis
2 For instance, see following experiments in the field: W. Edwards, "The Theory of Decision Making," Psychological Bulletin, 1954 and W. Edwards, "Behavioral Decision Theory," Annual Review of Psychology V, 1960
3 E.g. money pump argument (Ramsey, 1926; Davidson, McKinsey and Suppes, 1955)

There is usually a brief part in microeconomic and game theory textbooks, which deals with concerns of the preference axioms (e.g. Kreps, 1990 and 1988; Varian, 2003; Luce and Raiffa, 1957). The chapter of this thesis, which deals with the continuity assumption, is based on economist Rubinstein's work "Lecture Notes in Microeconomic Theory". Two works, which cover the theories of preference, will be stressed in this thesis: Fishburn's opus "Utility theory for decision making" and the article "Preference, Utility and Subjective probability" written by Suppes and Luce. Weinstein's article "Individual preference intransitivity" underlies the chapter that covers the transitivity axiom (Weinstein, 1968).

This thesis is a brief survey on how the preference axioms influence to applicability of the neoclassical microeconomic framework. My purpose is to handle axioms separately. It will be proceeded by gathering some of the essential concerns of the preference axioms from the literature. The arguments against the axioms will be critically evaluated. From the basis of these considerations, I shall outline implications to the applicability of the framework. Further, no attention is paid to the normative justification of the axioms. This is because the viewpoint of this thesis is of a hypothetic nature. Whilst the axioms are given normatively adequate, it will be studied decision situations in which the axioms might be violated.

The structure of this thesis is straightforward. There is a brief introduction part to the field of preference theory in economics. An advanced reader can ignore it. In the rest three chapters I shall address three axioms: the transitivity axiom, the completeness axiom and the continuity axiom, in respective order. Each of the tree chapters comprises three parts. First, I shall introduce the axiom. The second part, titled "Concerns of the axiom X", covers limitations of the axiom and situations in which the axiom might be violated. This part is the basis for the third part in which I shall draw implications from the second part. The third part can be regarded as a conclusion part and its purpose is to sketch restrictions for the applicability of the framework implied by the axiom in question. The thesis is intended for a reader who has basic knowledge about the set theory and calculus. The subject essentially involves mathematical structures and modeling. Therefore a mathematical formulation cannot be avoided. However, my purpose is to keep the mathematical jargon at minimum. That is why I mostly separate the technical stuff and place it in the footnote.

1 A brief introduction to algebraic choice theory for certain outcomes

1.1 Theories of Decision-Making in Economics

The field of decision-making is a vast area. A truly interdisciplinary subject has affiliations to many sciences such as psychology, applied mathematics and economics. The diversity of theories of choice is wide; different disciplines have developed own decision theories according to their needs. Especially in economics, there has been a huge demand for a theory of choice that describes the behavior of an economic agent. In microeconomics it is commonly referred to as the theory of consumer's choice. Game theory studies decisions made by a group, but the discipline is heavily based on the decision theory of individual.

The theory of preference is an overlapping subject and it usually gives psychological underpinnings for theories of choice. These differ in many respects. Whether to see preferences governed by probability mechanism? Does one treat outcomes of alternatives as certain or uncertain? Is it examined a stochastic case in which time is included into the model, or is it satisfied with a static model? A particular case, uncertain outcomes with probabilities, has been extensively studied by von Neumann and Morgenstern. They introduced a set of axioms on preferences, known as Von Neumann–Morgenstern axioms, in their well-known opus *Theory of Games and Economic Behavior* (von Neumann and Morgenstern, 1947). These axioms or a bit customized versions give a decision-theoretical foundation to the game theory (Luce and Raiffa, 1957). The most part of academic debate about this subject covers this special case. On the other hand, the case in which probabilities isn't included, is commonly gone through in microeconomic textbooks. The axioms presented in the consumer theory, are similar to VNM-axioms but easier to study due to absence of probabilities. Strictly speaking, the theory of consumer's choice is an algebraic choice theory for certain outcomes. The theory intends to describe how a consumer behaves in a hypothetical decision situation.

1.2 Framework

Let a set X be any set whose elements involves in some decision situation. These can be regarded as alternatives, political options or whatever. I adopt same notation as Kreps, so throughout the thesis I shall call this set a *choice set* (Kreps, 1988, p.3). The theory of consumer's choice takes a viewpoint that elements of a choice set are interpreted as consumption bundles. In that situation a choice set can be modeled as an n-dimensional vector, that is to say a choice set is a subset of $\mathbb{R}^n$. Moreover, there are two cases that determine the complexity of utility function proofs. Either a choice set is uncountable or it is a countable set[4]. In this thesis I shall, for the most part, study cases in which a choice set is a countable set. When dealing with the continuity axiom, considerations must be expanded to uncountable sets as well.

Preferences are modeled by a concept of binary relation. Preference relation is a binary relation that possesses specific properties. In a mathematical sense, a binary relation is an ordered pair or a collection of ordered pairs. It is essential to notice that a binary relation is defined on some set and it can be specified as a subset of the Cartesian product[5] for a set in question. This is essential because whether a binary relation possesses particular properties depends on a set in which it is defined. In context of preference relation that means: whether the preference relation satisfies particular properties depends on which choice set we are considering. There are a lot of well-known properties of binary relation and I have listed some of those[6] are relevant for the preference relation.

Linguistic meaning of preference relation goes as follows. If object x is weak preference related to y, denote by $x \succsim y$ or xRy, then it should be read as "x is at least preferred than y". Strict preference relation is denoted by $x \succ y$ or xPy and it should be read as "x is more preferred than y". In addition we can define a third relation called indifference relation. That is marked $x \sim y$ or xIy and it means the same as "subject is indifferent between x and y". The meaning of indifference relation is a bit ambiguous so it will be considered more carefully later.

There are two approaches how to deal with preferences. A common way is to define a weak preference relation as the primitive and the rest of relations are related via specific definitional connections. On the other hand, it is possible to begin with a strict preference relation. In this thesis it is followed in the former manner. It is recommendable to

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4 A set S is called countable if it has smaller or the same cardinality than the set of natural numbers. A countable set is either a finite set or a countably infinite set. For example, sets $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$ are countable sets.
Otherwise, a set is called uncountable. For instance, the set of real numbers is an uncountable set.

5 Suppose we have two sets X and Y. Then the Cartesian product $X \times Y$ is the set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$. And the Cartesian product of X with itself can be denoted as $X \times X$ or X^2.

6 A list of properties of a binary relation that will be mentioned through the text.
Reflexive: $\forall x \in X : xRx$
Complete: $\forall x,y \in X : xRy$ or yRx
Transitive: $\forall x,y,z \in X : (xRy \text{ and } yRz) \rightarrow xRz$
Negatively transitive: $\forall x,y,z \in X : [\neg(xRy) \text{ and } \neg(yRz)] \rightarrow \neg(xRz)$
Asymmetric: $\forall x,y \in X : xRy \rightarrow \neg(yRx)$

state definitional connections between different relations formally by means of logical connectives. This is due to avoid confusions (cf. Rechenauer 2008 and Quesada 2010). Table 1 presents definitional connections when a weak preference relation has been taken as the primitive.

$$xRy \leftrightarrow xPy \vee xIy \qquad (1)$$
$$xIy \leftrightarrow xRy \wedge yRx \qquad (2)$$
$$xPy \leftrightarrow xRy \wedge \neg yRx \qquad (3)$$

Table 1: Definitional connections when a weak preference relation has been taken as the primitive.

1.3 The axioms of choice

As mentioned before the primitive preference relation can be freely chosen between a strict and a weak version, given that definitional connections among preference relations are appropriately defined (Quesada, 2010; Fishburn, 1970). Once primitive preference relation has been selected, it can be proceeded in the following way. Either a weak preference relation can be postulated to be transitive and complete or a strict preference relation can be postulated to be negatively transitive and asymmetric. A binary relation that is transitive and complete is called a weak ordering. Because I shall take a weak preference relation as primitive, the axioms under study are following:

i. The completeness axiom
ii. The transitivity axiom

These axioms guarantee the existence of a real-valued utility function on a countable set. But generally the consumer theory is interested in applying the framework in the case in which the choice set is an uncountable set. This is due to the money and commodities are taken to be divisible units and to avoid the loss of generality. Unfortunately, these axioms don't yet guarantee the existence of a real-valued utility function on an uncountable set. One common counterexample is a lexicographic order; it doesn't have a real valued utility representation. That is to say, one more assumption is needed – the continuity of preferences. Gérard Debreu has proven that there is a continuous utility presentation if a preference relation is continuous (Debreu, 1954). This is known as the Debreu's Theorem. So, I shall also study:

iii. The continuity axiom

The subject of this thesis is confined into these three axioms. There are several other assumptions on preferences presented in economic writings such as nonsatiation, monotonicity or convexity. Function of these assumptions is only to facilitate formal analysis. However, there is one assumption that involves relating one's preferences and his choice behavior. This assumption describes according to which the agent makes a choice. It has been suggested that this assumption should be treated as truism (Weinstein, 1968). Nor shall I deal with it, but it is needed to be posed for subsequent purposes:

iv. The subject will always chose an alternative the one he excepts will leave him the more preferred position[7].

2 The transitivity axiom

2.1 Description

The first axiom, transitivity, is the most essential but also the most controversial axiom. It is a key property of the normative choice theory. Transitivity formally captures the intuition behind the notion of order. If there were no preference ordering among alternatives, it would be difficult to imagine a preference based choice theory that has reasonable content. "This assumption [transitivity] is necessary and essentially sufficient for the representation of preference by an ordinal utility scale - -" (Tversky and Kahneman, 1986, p. 210). On the other hand, this is the most disputed postulate. There are countless articles that cover how transitivity fails to describe individual preferences.

A transitive binary relation comprises a following property. If an element a is related to an element b and an element b is related to an element c, it holds that a is related to c. Intuitively, it's a some kind of a chain property between elements. For instance in the context of preference relation, suppose that there are three alternatives: "relaxing", "working" and "sleeping". Transitivity assumption says that if an agent prefers relaxing to working and working to sleeping, it has to be the case that he prefers relaxing to sleeping. The transitivity axiom is regarded as consistency property of an agent behavior (e.g, Luce and Raiffa, 1957; Deaton and Muellbauer, 1980, p. 27).

[7] Assume that the choice set is a set A. Then, a set of chosen alternatives is:
$c(A) = \{x \in X \mid \forall y \in A : x \succsim y\}$ (The best choice connection)
(Kreps 1990, p.25; Stanford encyclopedia of philosophy, 2011)

2.2 Concerns of transitivity

This section is mostly based on the Weinstein's article "Individual preference intransitivity" (Weinstein, 1968). Weinstein has been capable to encapsulate some essential concerns of intransitive preferences. He divides causes of observed intransitivity into five categories. The first category includes causes that are "true irrationality of a clinical nature". For instance, conflicts between parts of the Freud's structural model of the psyche and conflicts with the individual internal value systems are counted in this category. From the point of view of the issue, delving into this category would not be appropriate, because it would end up in discussions too deep in the physiological analysis. I rephrase the second category by words "inadequate experimentation settings". This category contains failures such as "a lack of communications between experimenter and subject" and "the experimenter asking the wrong question". It should be mentioned that it also contains dynamic inconsistency, which cannot be necessarily classified under a category of inadequate experimentation setting. The third category is a trivial case. Non-allowance of indifference relation might cause intransitivity but this is not a concern of the economic theory. Namely, traditional theories of decision-making in economics accept the possibility of indifference relation. The fourth category involves in concerns about the JND-assumption. Weinstein admits that this concern is marginal one. It's closely related to the continuity axiom; therefore I shall treat it in Chapter four. Weinstein describes the last category by words "rational intransitivity due to the high cost holding a preference ordering". I shall emphasize this category and end up questioning its existence. Weinstein views that the economist should show concern to the causes described in the first and the fifth category. (Weinstein, 1968).

At the end of the section I shall bring out few other causes of intransitivity. The framing effect is commonly recognized as cognitive bias in psychology. The presence of the effect might cause intransitive behavior. Furthermore, there are several special situations in which transitivity might be violated.

Weinstein includes observed intransitivity caused by "the subject's changing tastes" into the second category. Intransitivity induced by decision maker's dynamic inconsistency is not a direct concern of static theories of preference. Dynamic inconsistency is a situation where the subject's preference changes over time setting off inconsistencies among preferences at different points in time. For instance, imagine an experiment in which it is examined preferences of an individual for fashion jeans. If the time period is long enough, it will be unrealistic to assume that the individual has a static preferences during that time period. In that case, the experiment suffers from dynamic inconsistency. The theories of preference that "include time parameter" are classified as stochastic theories of preference (e.g. Luce and Suppes, 1964). Stochastic theories of preference intend to describe preference changes over time, therefore dynamic inconsistency is not a concern of the static theories of preference – preferences are taken as exogenous at a static point of time. It seems quite obvious that assumptions about invariable preferences should never be made, because it is a default that preferences are not transitive over time. Hence, this aspect just should be taken into account in empirical experiments. Edwards plausibly argues that the assumption of "constancy of tastes over the period of experimentation" is necessary in empirical experiments in order for the experiment to be meaningful (Edwards, 1954, p. 46). Furthermore, one should bear in mind that the framework is quite constrained due to the assumption of static preferences.

Weinstein suggests that intransitive results in May's experiment may be caused by boredom or lack of the reward (Weinstein, 1968, p. 340). This is an example of a situation in which subjective utility and objective utility conflicts and it describes a difficulty to arrange appropriate settings for empirical preference experiments. I shall illustrate the problem by proceeding with May's experiment[8] (May, 1954). There is need for background assumption in order for the conclusion of May's results to be correct. For example, it has to be assumed that a choice set, which the subject is reflecting, is a set of "hypothetical marriage partners". Otherwise one cannot construct an argument against transitivity. I shall provide another potential choice set that the subject is actually reflecting. Suppose that the choice set is a set of two elements "choose an alternative at random" and "try to reflect yourself in an actual choice situation and make your choice in accordance with it". I argue that if the subject strongly avoids deliberation of the test questions, the choice set I presented will be a more potential one. There is a lesson for us; a choice set may deviate from the questionnaire of the experiment. A possible response to this kind of a problem is proposed by Anand. He sees that "the violations of transitivity can be removed by redefining the choice primitives", but continues by questioning whether "transitivity is a feature of behavior or of language" (Anand, 1993). Since this problem of setting appropriate "choice primitives" is not easily solved, one should be careful with his inferences. The experimenter should not ask "the wrong question".

[8] In the experiment, college students were asked to choose prospective marriage partners in accordance three criterions: appearance, wealth and intelligence.

Weinstein deals with the category five in chapter titled "Rational intransitivity" in his article. He states the cause of the fifth category is "pregnant with possibilities for the economist". In my opinion there is a possibility for misunderstanding. Weinstein argues the existence of this fifth category based on the passage of Edwards's writings: "*It is conceivable, for example, that it might be costly in effort (and therefore negative in utility) to maintain a weakly ordered preference field. Under such circumstances would it be ´rational´ to have such a field?*" (Edwards, 1954). Recall that a choice set is a set of alternatives (or prospects or outcomes, etc.). The theory of choice is based on the assumption that a decision maker chooses the most preferred alternative. The notion of utility is a secondary concept, the utility function can be derived under certain assumptions and it reflects a satisfaction of preferences. Put differently, utility is a measure of preference satisfaction. So strictly speaking, the subject maximizes preference satisfaction function. Against this viewpoint, how does a statement "it might be costly in effort (and therefore negative in utility) to maintain a weakly ordered preference field" sound like? I assert that *a category mistake* has been committed in the latter statement[9]. It can be regarded that "preference field", "an effort [negative utility]" and "to maintain something [alternative]" belongs to different categories. First, even though the subject would be able to select his preferences, he will not be able to select *properties* of his preferences. That is, the theory assumes that the subject's preferences satisfy certain properties: transitivity, completeness… whether the preferences are transitive or not but is not matter of how the subject chooses it to be. Therefore the alternative "maintain of a weakly ordered preference field" is not an acceptable alternative. In other words, you cannot treat the former alternative as acceptable and at same time use concepts such as utility to construct an argument against the theory. There will not be such a concept as utility if properties of preferences are not what the theory assumes. Second, due to "utility" and "preferences" belongs to different categories Edward's statement leads to a contradiction. Consider two propositions below:

(1) *A subject maximizes a measure, which is deduced from a set of assumptions*

(2) *If the set of assumptions holds, it could to be the case that the subject is not maximizing the measure.*

[9] The term "category-mistake" is introduced in the book The Concept of Mind (Ryle, 2002, p. 327).

The proposition (1) is true on the grounds of the Debreu's Theorem. If the preference assumptions[10] hold, then the maximizing property of the measure (utility) will be true. How about the second one? The content of the proposition (2) is the same as Edward's statement has in disguise. If the subject chooses an alternative, which yields "negative utility", then he won't maximize his utility given that there is at least one alternative, which yields neutral utility. The maintaining of a weakly ordered preference field yields negative utility and its complement alternative "not to maintain a weakly ordered preference field" yields at least neutral utility, otherwise above-referred passage lost its meaning. So, the maintaining of a weakly ordered preference field is not maximizing utility. Preferences of the subject obey the set of axioms if and only if he has a weakly ordered preference field. To get proposition (2), just combine the last two sentences. Now, to be more rigorous let us denote proposition formulas by proposition symbols A and B:

A = "The set of axioms $\{a_1,a_2,a_3,a_4\}$[10] holds."

B = "The subject maximizes the measure (utility)."

The proposition (1) says that A implies B. In contrast, the proposition (2) states that it can be the case that A implies a negation of B. So, if a set of assumptions holds, then Edward's statement will be false[11]. If a set of assumptions does not hold, then there won't be such thing as utility and the statement means nothing. Hence, I see that Weinstein's fifth category is not a concern of the transitivity axiom. The crux of the problem is that the measure, in other words a utility, is a secondary concept with respect to the preference (or preference assumptions). Further, I regard that Edwards has psychologically appropriate reasoning but he mess up with the economic concepts (a utility, a preference) when constructing his argument.

The framing of the option comprises all relevant aspects of formulating a choice problem. For instance, these aspects can be "the language of presentation", "the nature of the display" or "the context of choice" (Tversky and Kahnemann, 1986). The main idea is that the settings around the option matters, not solely the option itself. It follows that the subject's preferences for an alternative might vary in different framings. Kreps introduced "the framing

[10] The set of assumptions presented in the first chapter. Recall the assumptions I – IV.

[11]
$$\dfrac{\dfrac{A \to \neg B \qquad [A]}{\neg B}\to E \qquad\qquad \dfrac{A \to B \qquad [A]}{B}\to E}{\text{contradiction}}\wedge I$$

of a particular choice" as a concern of the asymmetric property of a strict preference relation (Kreps, 1990, p. 20). Tversky and Kahneman state that "the axiomatic analysis of the foundations of expected utility theory" reveals four assumptions besides the assumptions of comparability and continuity. One of these assumptions is the assumption of invariance – "different representations of the same choice problem should yield the same preference". (Tversky and Kahnemann, 1986, p. 210–211.) The violation of this assumption is evident by the framing effect. Framing may create a situation in which between two alternatives, the subject prefers both alternatives to another at the same time. Via definitional connection (3, Table 1) this produces logical impossibilities. So, one can be sure that problems aren't avoided nor in the case a weak preference relation is taken as primitive. The existence of the framing effect is not really a concern of the completeness assumption but this is surely a concern of the transitivity assumption. To see this, consider a situation in which the subject should order three alternatives, let's say a, b and c, on basis of his preferences. For each alternative there are two different ways to present an alternative, I mean two different framings: framing one and framing two. Let us assume that the subject will be indifferent among alternatives, if alternatives are framed in the same way. But for each alternative, he strictly prefers alternative framed as in framing two. Now, suppose that the experimenter asks the subject to choose:

1. Between a and b, both framed as in framing one. 2. Between b and c, both framed as in framing one. 3. Between a and c, a is framed as in framing one but c is framed as in framing two.

The transitivity will be violated, if questions are posed in this way[12]. So, it should be noted that the framing effect might produce an intransitive behavior.

The subject may have intransitive preferences when he is maximizing the probability of winning. This kind of paradox is known as "a statistical paradox" (Anand, 1993, p. 344). There are several examples of hypothetical decision situations in which the paradox can be obtained. Perhaps, the most well-known is the game played with *intransitive dice*, which appears in many writings (e.g. Blyth, 1972; Savage, 1994; Anand, 1993). Consider the three six-sided dice having following numbers on their faces a = (3,3,5,5,7,7), b = (2,2,4,4,9,9), c = (1,1,6,6,8,8). Two players are playing the game, where each throws their dice and the

[12] By the first and the second questions it holds that a~b and b~y, which imply a≿b and b≿c. By the third question it holds that c≻y. This of course implies c≿y and not a≿c. So, it doesn't hold that: a≿b and b≿c imply a≿c.

player with the highest number wins. At the beginning of the game, some third party selects two dice from the above-mentioned three dice. The first player chooses a dice and leaves the alternative for the second player. Now, if the outcomes of the dice are looked, it will be easily seen that the preferences of the first player are: a≻b, b≻c, and c≻a. This implies intransitivity of a weak preference relation[13].

As I mentioned earlier, if choice primitives are defined in a different way, the problem will be avoided. Depending on which two dice were selected, the remaining sets of alternatives are {a,b}, {a,c}, {a,d}. So, why not define alternatives as: a´ = "select a when {a,b}",b´ = "select b when {a,b}", c´ = "select a when {a,c}", d´ = "select c when {a,c}", etc.. ? In that case, the first player prefers a´ to b´, d' to c´, etc.. and the preference pattern is transitive. The case reminds an extensive-form game defined as follows. At the start of the game chance chooses dice x with probability 1/3. Then the first player chooses a dice from one of the two dice and the remaining dice ends up to the second player. The payoffs are: one if the player wins with higher probability and zero otherwise. The optimal strategy for player one is strategy (bca). If the choice situation is modeled in this way, the argument will be inadequate. So, the questions is "Why not model the choice situation in this way?" The game is described in Figure 3.

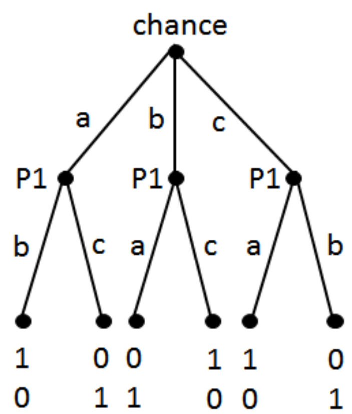

Figure 3: An extensive game with perfect information and chance moves.

13 (aPb∧bPc∧cPa) → [(aRb∧bRc∧cRa)∧(¬aIb∧¬bIc∧¬cIa)], so combine: aRb∧bRc → cRa∧¬cIa, which means aRb∧bRc → ¬aRc.

2.3 Limitations of the framework induced by the transitivity axiom

In order for the transitivity axiom to be valid, there must be made an assumption that preferences are invariable during the period studied. Therefore one should be aware that potential preference changes over time are ruled out.

If "choice primitives" in the choice set don't correspond to actual alternatives the subject is reflecting, it will be likely that intransitive behavior is exhibited. This means that a user of the framework should check whether the choice set is realistic when bearing in mind the modeling situation. This kind of problem emerges when "a statistical paradoxes" are studied. The game played with intransitive dice shows that intransitive behavior may occur when the subject aspires to maximize the probability of winning.

The framing effect was originally stated against other axioms of rational choice (Tversky and Kahnemann, 1981 and 1986; Kreps, 1990). Nevertheless, it is possible to convert the framing effect against the transitivity axiom. Converting is easy, because the framing effect causes "true irrationality" – the subject strictly prefers both alternatives at the same time. Thus, when building the model, a correct framing should be incorporated into the model. I mean the framing that the subject actually faces in a particular economic environment and other framings of the same alternative should be added to choice set as different alternatives.

The Weinstein's fifth category is not a concern of the transitivity axiom. Given that the notion of utility and preference are defined as economists these define, it will be absurd to insist that: "it might be costly in effort (and therefore negative in utility) to maintain a weakly ordered preference field". Showing that the statement of the fifth category is either false or meaningless depending on whether the preference axioms are true, I conclude that the former statement is not a concern of the transitivity axiom. The crux of the problem is that the notion of utility is secondary with respect to the notion of preference.

3 The completeness axiom

3.1 Description

Since the completeness axiom with the transitivity axiom guarantee the existence of a real-valued utility function on a countable set, the completeness axiom can be regarded as the second most essential assumption of the rational choice theory. A complete binary relation is also called total, linear or connected. The completeness axiom also has more names such as the connectivity assumption or the comparability assumption. The last one describes well what the axiom is in character. Namely, the completeness of a weak preference relation implies that an agent is perfectly able to decide whether he prefers something to something or be indifferent among them. An economic agent who obeys this axiom can't express a statement such as "I don't know" or "I don't take a stance on it".

Completeness of a binary relation indicates that there is a relation between each element. That is to say, between two elements x and y, it must be the case that either x is related to y or y is related to x. In the case of weak preference relation, it can be interpreted as follows. For each alternative, either it is weakly preferred to some other alternative, or some other alternative is weakly preferred to it. It should be noted that "or" is interpreted as inclusive or, so it is possible that both "x is at least preferred than y" and "y is at least preferred than x" are true at the same time. This special case is defined as indifference relation between x and y. Since completeness implies reflexivity, it should be also considered the plausibility of reflexivity property in weak preference relation. Reflexivity means that each element is related to itself. The theory assumes that each alternative is as at least preferred as itself.

A distinction should be made between the understanding of consequences and the ability of decision-making. Preference theories for certain outcomes assumes that there is no uncertainty which outcome will occur if some alternative is chosen – the chosen alternative is the same as the obtained outcome. The subject's comparability difficulties, caused by uncertainty about how likely implemented action leads to specific outcome, isn't a concern of the theory. Dealing with the theories of preference that takes in to account uncertain outcome, the problem can be formulated more precisely. Savage's version of the expected utility theory treats probabilities as being subjective, like a subject's beliefs. In that case we allow that selected action leads to an outcome with specific probability (Savage, 1954). The subject is dubious about probability, but if the subject knows that probability, he won't hesitate a moment which one he chooses.

3.2 Reflexivity

Every complete binary relation is also reflexive[14]. Nevertheless, reflexivity is presented as a "mathematically necessary" axiom in some writings (Deaton and Muellbauer, 1980, p.26). Since the completeness is postulated, reflexivity is not necessary anymore in the mathematical sense. Certainly, reflexivity is still necessary in the sense that a weak preference relation has to be reflexive in order to be a total order. But we are talking about *axioms* and "the reflexivity axiom" isn't necessary, because it is implied by another axiom. Hence, I regard that it is misleading to say that reflexivity is a mathematically necessary axiom. Axioms that have a crucial role in the model and cannot be deduced from other axioms, are necessary. Otherwise, these are called unnecessary. This is not to say that we don't have to reflect plausibility of reflexivity. To do with axiomatic system, one shouldn't only examine axioms themselves, rather implications also induced by axioms. Plausible axiom with questionable implications should be rejected. Fortunately, reflexivity property of weak preference relation doesn't seem to produce difficulties. In general, reflexivity is regarded as a trivial case and it is not a truly concern.

Reflexivity of indifference relation is embedded in the meaning of language as follows. Every state is identical with itself. Two identical states induce identical stimulus, so there cannot be discrimination among each state. Since there cannot be discrimination, individual cannot prefer one to another. If we ignore non-comparability possibility, it has to be the case that the subject is indifferent between two states. By definition (2, Table 1) it follows that a weak preference relation is reflexive. It is a bit confusing to talk about comparing something to itself and that stretches far from meaningful language. For this reason, it can be suggested that reflexivity of weak preference relation should be based on reflexivity of indifference relation.

3.3 Concerns of comparability

Aumann sees that "the completeness axiom is perhaps the most questionable [axiom]". He proceeds by writing that from the descriptive viewpoint it is inaccurate and it is not plausible

[14] Assume that x=y. By completeness it holds that xRx or xRx. Therefore, for all x in X it holds that xRx.

“even from the normative viewpoint”. He thinks that “rationality” doesn’t demand that an individual should be able to make “definite comparison between all possible lotteries”. (Aumann, 1962.) The common wisdom is that the completeness axiom is invalid from the descriptive viewpoint. But claiming that the completeness assumption is not valid even from the normative viewpoint is at least a bit of a controversial claim. Varian writes in his widely used microeconomic textbook: “*The First axiom, completeness, is hardly objectionable, at least for the kinds of choices economists generally examine*” (Varian, 2003). This statement can be interpreted as claiming that the completeness axiom is a normatively appropriate assumption in economic applications. One way to approach the problem is to ask whether there are potential economic applications in which the completeness assumption is inappropriate. Three choice categories are proposed that might violate the completeness axiom.

Varian proceeds by suggesting that there may be a situation “involving life or death choices where ranking the alternatives might be difficult“ (Varian, 2003). Undeniably, one might be confronted with choice situations that involve huge changes in one’s life. For instance, it can be hard to choose between health and wealth. Which one is preferred: to be physically lame but rich or to be able-bodied but extremely poor? Too extreme choices may disturb a preference formation.

There is another kind of choice that is closely related to the previous choice category. The choices needing ethical consideration might not be comparable. A concrete example could be a choice situation that a doctor might encounter. Consider a situation in which a woman has a difficult parturition and the doctor is forced to choose whether to make an abort or not. This category comprises choices that involve ethical judgments.

The third choice category contains choices that possess different attributes whereby a subject constitutes his preferences. For instance, imagine that a subject has to give a vote for a candidate between two candidates; let say Smith and Watson. The subject prefers Watson to Smith in the sense Watson is more liberal. On the other hand, the subject prefers Smith to Watson in the sense Smith is more fiscally responsible. The subject might have difficulty to form a judgment about which one he prefers. If alternatives contain “different attribute scales”, it might be difficult to constitute preference. Psychologists call this phenomenon “a multidimensional phenomenon”. (Luce and Raiffa, 1957, p.25).

The situations, in which a subject is indifferent between two alternatives, are of special character. There doesn’t normally exist a clear threshold when the subject changes his position from being indifferent to preferring something to something. Framing and that kind of things affect crucially the conclusion of the subject's decision process when he's pondering whether he is indifferent or not. This concern should be taken to account in empirical experiments. Suppes and Luce crystallize this idea in the chapter which addresses the nature of preferences: *" - - when a subject exhibits some inconsistency in his choices in a preference experiment, we automatically attribute it to an ambivalence about the worth of the outcomes, not to his inability to tell which outcomes are associated with which stimulus-response pairs - - "* (Suppes and Luce, 1965, p.254). The same problem arises when we are examining a situation in which a subject is considering how he reacts indefinitely small changes among the alternatives. This issue is closely related to the concerns of the continuity axiom and therefore I shall more carefully deal with it in Chapter four.

There will arise several concerns if the completeness axiom is dropped. First, it's quite clear that the most common existence proofs of the utility function are no longer valid. That means rejection of the Debreu´s theorem and the Von Neumann-Morgenstern utility theorem. There have been developed utility theories without the completeness axiom (Aumann, 1962). I won't treat them but instead, I shall illustrate arising difficulties in the case we only assume reflexivity instead of completeness. The proof of the existence of utility function is quite straightforward in the case that the choice set is finite[15]. There exists a utility function that preserves preference order, that is $x \succsim y$ implies u(x) > u(y). But how about conversely, does u(x) > u(y) imply $x \succsim y$? No, since relation is now a partial order, converse isn't true. This kind of situation is illustrated in Figure 1. The relation is reflexive, transitive and antisymmetric, so it is a partial order. Let us suppose that the utility function is in the same form as used in the proof[16]. We have u(e) = 1 and u(c) = 2, but it isn't true that $c \succsim e$. In conclusion, a utility function that preserves preference order still exists, but from values of a utility function cannot be deduced to preferences.

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[15] This is a modified version (only reflexivity is assumed) of Rigotti´s proof (Rigotti, 2014):

$Let\ adopt\ notation\ R(x) \equiv \{y \in X \mid xRy\}\ and\ define\ u(x) \equiv card(R(x)).$

$\underline{First\ suppose\ that\ xRy}.\ \forall z \in R(y)\ it\ holds:\ yRz.\ So,\ xRy \wedge yRz \rightarrow xRz\ by\ transitivity.$

$Therefore\ z \in R(x). We\ have\ shown\ that\ R(y) \subseteq R(x). This\ implies\ card(R(y)) \leq card(R(x)).$

$By\ definition\ u(y) \leq u(x).\ \underline{Second,\ suppose\ that\ xPy}.\ It\ holds\ xRy \vee \neg yRx\ by\ definitional\ connection\ (3).$

$On\ the\ basis\ of\ above\ it\ holds\ (R(y) \subseteq R(x)) \wedge (x \notin R(y)). So,\ it's\ true\ that\ R(y) \cap \{x\} = \emptyset.$

$And\ it\ holds\ xRx$ **by reflexivity**. $That\ implies\ x \in R(x).$

$Finally,\ R(y) \cup \{x\} \subseteq R(x) \rightarrow card(R(y) \cup \{x\}) \leq card(R(x)) \rightarrow card(R(x)) + 1 \leq card(R(x)) \rightarrow u(y) + 1 \leq u(x) \rightarrow u(y) < u(x) \quad \square$

[16] We defined u(x) ≡ card({y∈X | xRy})

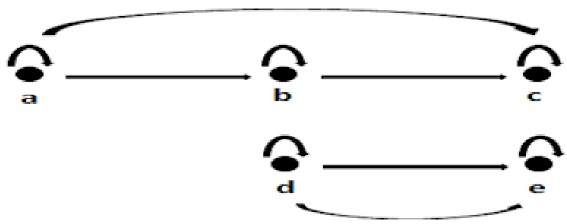

Figure 1: A choice set contains five elements and arrows depict preference relations. Notice that none of elements a, b and c is related to elements d or e.

A second concern of non-comparability possibility involves breaking other axioms. Namely, presence of non-comparable alternatives implies virtually inevitably intransitivities in preference ordering. Luce and Raiffa suggest that: "*intransitivities often occur when a subject forces choices between inherently incomparable alternatives*" (Luce and Raiffa, 1967, p.25). The idea is intuitively quite clear. For instance, consider a situation where little Patrick is contemplating which candy he should choose. Let us suppose that he is able to say which one he prefers between each candy. But there is one exception. Patrick isn't able to say how he deals with acrid lemon candy. The only thing that he can say is that red bubblegum is strictly better than acrid lemon candy. Unfortunately, Patrick´s preference ordering doesn't satisfy transitivity property in any case in which red bubblegum isn't the most delicious one. Figure 2 illustrates how incomparability violates the transitivity axiom.

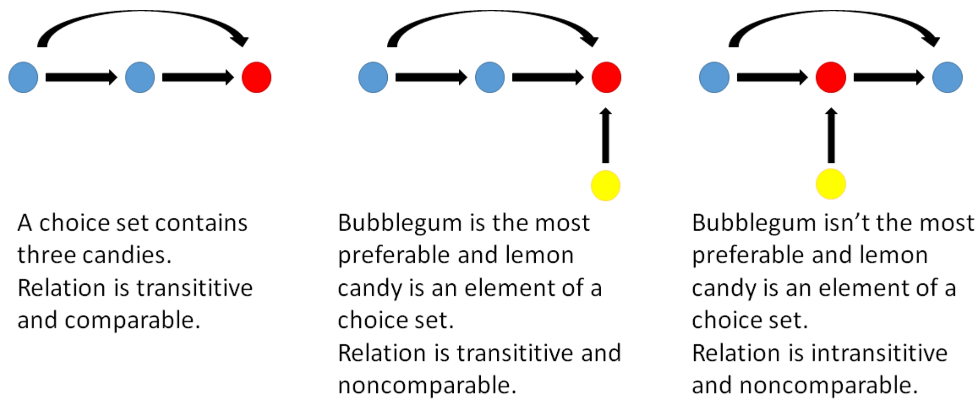

Figure 2: Three cases are illustrated. Arrows depict preference relations.

This possibility should be recognized. Consider an experiment in which it is tested whether individuals have transitive preferences. The experiment is conducted by giving a questionnaire to a subject. Every question has four alternatives to answer: "I prefer A to B", "I prefer B to C", "I am indifferent between A and B" and "I can't say". The possibility to answer "I can't say" might distort a result of the experiment, because of the reason just discussed. Even if the alternative "I can't say" is discarded, the problem may still exists. That is because the subject might genuinely think that he can't say whether to prefer or not. In this case the subject will likely choose an alternative "I am indifferent between A and B", which is disinformation. Yet, the result of the experiment might unjustifiably support the notion that the individual has intransitive preferences.

3.4 Limitations of the framework induced by the completeness axiom

The completeness axiom restricts the applicability of the framework. The choice set shouldn't contain elements that are not comparable by subject. It should be checked does the choice set contain inherently incomparable alternatives. If there are potential ones, those should be removed from the choice set. One essential reason for that is the propensity of non-comparable alternatives to produce intransitivities.

In practice, the completeness axiom rules out "extreme" choice situations. What is it meant by describing a choice situation as "extreme"? Three kinds of choice situations are proposed to classify as "extreme" ones. If the choice set contains alternatives that involves: (i) Enormous changes in one's life, (ii) Profound ethical evaluations or (iii) "Different attribute scales".

4 The continuity axiom

4.1 Description

As I mentioned in Chapter one, the continuity axiom is redundant when the choice set is taken to be a countable set. This axiom is only needed when expanding considerations to uncountable sets, for instance the choice set can be regarded as the n-dimensional Euclidean space. The natural question is: "Why is it needed to expand considerations to uncountable sets?" The answer is purely a technical one. In order for techniques of calculus and mathematical programming to be applicable, it will be necessary to deal with uncountable sets (e.g. $\mathbb{R}$ or $\mathbb{C}$). The transition from countable sets to uncountable sets does not enlarge the range of actual modeling situations. From this viewpoint, there is never need to treat the choice set as an uncountable set instead of treating it as a countable set. The set of rational numbers is a countable set and the difference between the set of real numbers and the set of rational numbers is only mathematical. Intuitively, both sets compromise all numbers, so that there is “infinitesimal small change” between two adjacent numbers. So, it should be kept in mind that all the time we examine cases in which the choice set is an uncountable set.

There are several ways to define a continuity property of a preference relation. I shall adopt the manner in which Rubinstein, as many other authors, introduced the continuity property (Rubinstein, 2006). Let a sequence $\{(x_n,y_n)\}$ be a sequence of pairs of elements in a choice set X satisfying $x_n \succsim y_n$ for all n and $x_n \rightarrow x$ and $y_n \rightarrow y$. Then it has to be the case that $x \succsim y$. As a mathematically unorientated reader might see, this axiom is slightly more technical than others. So, I shall clarify a content of the axiom by an example. Assume that $(x_n)_{n \in \mathbf{N}}$ is a sequence of different amounts of money, let's define $x_n \equiv 1/n + 1$. Let a sequence $(y_n)_{n \in \mathbf{N}}$ be identical with the latter except y_n is defined $y_n \equiv 1/n$. Given that, the subject prefers more money to less, it holds $x_n \succsim y_n$ for all n. Limits are $x_n \rightarrow 1$ and $y_n \rightarrow 0$. Now, it can be seen that continuity property holds, because it is true that $1 \succsim 0$ by the latter assumption.

The continuity assumption does not have a clear intuitive content. I shall propose a tentative suggestion for how to interpret the assumption by decomposing the definition into three parts and make inferences based on it. The definition contains three conditions.

(1) The subject is able to perceive infinitesimally small changes in alternatives (consumption bundles).

(2) Furthermore, the subject is able to rank these alternatives (bundles) according to his preferences.

(3) The choice set contains the limit values of both sequences (of consumption bundles).

The condition (1) follows from the fact that the formulation of examined sequence $(x_n)_{n \in \mathbf{N}}$ can be arbitrary. If the subject isn’t able to discriminate a difference between two successive elements, which is $\Delta x_n = x_n - x_{n-1}$, the definition of continuous preference won’t be meaningful. How could there be preference without discrimination of two alternatives? Notice that this difference Δx_n can be infinitesimally small. The condition (2) is only a literal rephrase for the mathematical condition; $x_n \succsim y_n$ (or $y_n \succsim x_n$) for all n. The last condition (3) is the vaguest. It might be difficult to come up with a situation or a preference pattern that contains “limit value of a sequence” so that limit values are relevant from the point of view of the choice problem. There exists a classical example, which will be presented soon. Heretofore, it’s sufficient to conclude the last condition (3) states that the choice set contains limit values of both sequences, that is $\lim_{n\to\infty} x_n = x \in X$ and $\lim_{n\to\infty} y_n = y \in X$. Now, if these three conditions (1–3) hold, then:

(S) It has to be the case that the limit value of the preferred sequence (that is x) is preferred to the limit value of the unpreferred sequence (that is y).

Next, I shall consider the situation in which this statement (S) will be violated. Violating of the statement (S) implies violating the continuity axiom.

4.2 Concerns of continuous preferences

Debreu has shown that the lexicographic preferences do not have a real valued utility representation (Debreu, 1954, p.164). Unfortunately, the lexicographical preference pattern is a plausible procedure for preference formation. The logic behind the name "lexicographical order" comes from how words are ordered in a dictionary, based on the alphabetical order of their component letters. I shall illustrate the lexicographical order by an example instead of presenting the formal definition[17]. Consider an unsophisticated consumer who selects a wine bottle based on two criterions; the price and the label. He doesn't care much about the label, but the price is important. The price is the primary criterion with respect to the label. If the price is the same between two wine bottles, the label matters. But even minimal change in price makes the consumer select the cheaper one, no matter how elegant the label is. In this case, it is said that the consumer has lexicographical preferences.

The lexicographic preferences do not satisfy the statement (S), even though the conditions (1–3) hold. For instance, let's define $x_n \equiv (1/n, 1/n)$ and $y_n \equiv (0, 1+1/n)$. So, it follows that $x_n \to x = (0,0)$ and $y_n \to y = (0,1)$. Given that, the subject has lexicographical preferences (e.g. the first component is preferred to the second component) and more is preferred to less, it holds $x_n \succsim y_n$ for all n. But, it is not true that $x \succsim y$, because $(0,1) \succsim (0,0)$. So, the lexicographic preferences are ruled out by the continuity axiom.

Condition (1) leads us to the issue known as "JND assumptions" (e.g. Luce and Suppes 1964, Weinstein 1968). The notion of the just-noticeable difference (JND) is originally a psychological term. The just-noticeable difference (or difference threshold) is the smallest change in a stimulus which a person can detect half of the time. It is a measure of the subject's sensitivity to discriminate among different stimulus. In the context of the current issue, it can

[17] Assume two partially ordered sets X and Y. The lexicographical order on the Cartesian product X × Y is defined as $(x_1,y_1) \succsim (x_2,y_2)$ if and only if $x_1 \succsim x_2$ or ($x_1 \sim x_2$ and $y_1 \succsim y_2$).

be regarded as the subject's sensitivity to discriminate among different alternatives. Thus, a difference between two successive elements, that it's to say $\Delta x_n = x_n - x_{n-1}$, can be regarded as a JND of the alternatives. The problem is, of course, that a JND can't be extremely small. This reflects the fact that the axiom of continuity is of a technical nature.

As I mentioned in Chapter three, concerns of the JND are also related to other axioms. Complete and transitive weak preference relation implies transitive indifference relation. Armstrong (1951) argues, plausibly enough, that a given alternative A may be indifferent to a second alternative B, B may be indifferent to a third alternative C and still A is preferred to C (Luce and Suppes, 1964, p.279). This is due to the individual has limited capability to perceive physical stimulus and make a clear judgment of preference. So, the JND isn't only a concern of the continuity axiom but also a concern of other axioms.

4.3 Limitations of the framework induced by the demand of continuous preferences

Since transitive and complete preference relation is sufficient to guarantee the existence of utility function on a countable set, the continuity axiom is redundant when the choice set is taken to be a countable set. For technical reasons, it is convenient to treat the choice set as an uncountable set, in which case the continuity of preference is necessary. The definition of continuous preferences conflicts with the intuition. An example of that is the demand of the subject's infinitesimally small JND among alternatives. Thus, it is reasonable just to study implications of continuous preferences instead of trying to interpret its content. One such a clear implication is that the continuity axiom rules out certain types of preference structures; types which not satisfies the continuity property. For an example the lexicographic preferences are ruled out.

Conclusions

Two aspects complicate the analysis, especially when evaluating arguments constructed by authors from other disciplines than economics. First, besides explicit assumptions the theory postulates, there are also some implicit ones. For instance, preferences are taken to be exogenous, that means the theory assumes that the subject cannot choose his preferences. It is possible that an author construct a plausible argument against an axiom but due to it omits some implicit assumption, the argument is not targeting correctly. Second, the terminology sometimes leads to misunderstanding. Especially, the notion of utility causes confusion because of its historical use. How the concept is used in economic discourse differs quite a lot from its use in normative ethics (see *utilitarianism*).

It turned out almost an impossible task to treat the axioms completely independently. Concerns that threat plausibility of an axiom may be also a concern of other axioms. Three examples were presented. The existence of non-comparable alternatives may imply an intransitive preference pattern. The just-noticeable difference is a concern of the continuity axiom but also a concern of the transitivity and completeness axiom. The framing effect bothers several assumptions of rational choice.

In summary, I shall list restrictions for the framework implied by three axioms studied. The framework is not applicable to model "extreme" decision situations. Three this kind of situation are proposed: a situation in which involves: (i) Enormous changes in one's life, (ii) Profound ethical evaluations or (iii) Alternatives that contains different "attribute scales", which may be problematic for a subject. The choice set should not contain alternatives that are not comparable among each other. An assumption of static preferences should be made because the framework excludes time parameter. If there is a time difference between two choices in actual choice situation, a user of the framework should consider that the time difference is short enough not to cause preference changes. One should be careful when considering relevant alternatives to include into the choice set. After all relevant aspects have been taken into account, do actual alternatives, which the subject is reflecting, correspond to elements in the choice set? The framing effect should be noticed – the correct framing should be incorporated into the model. Certain types of preference structures are ruled out, for instance the lexicographic preference pattern.

The purpose of this thesis is only to outline the applicability of the framework. Immediately, it should be noted that the above-mentioned list of restrictions is not comprehensive; some issues[18] were omitted and the concerns addressed in this thesis need further analyzing. Above all, it is just a matter of what restrictions we impose on a choice set. Decision situations we are going to model are incorporated into the framework via elements of a choice set. That means, "inapplicable" elements of a choice set reflect the restrictions of the framework. More preciously: a choice set cannot contain elements that violate the preference axioms, and these "inapplicable" elements express restrictions of the framework.

[18] e.g. *The anchoring effect* or *the Condorcet's paradox* when modeling collective preferences

References

Anand, Paul. 1993. "The Philosophy of Intransitive Preference." *The Economic Journal* 103 (417): 337–46. doi:10.2307/2234772.

Aumann, Robert J. 1962. "Utility Theory without the Completeness Axiom." *Econometrica* 30 (3): 445–62. doi:10.2307/1909888.

Deaton, Angus, and John Muellbauer. 1980. *Economics and Consumer Behavior*. Cambridge ; New York: Cambridge University Press.

Debreu, G. 1954. "Representation of a Preference Ordering by a Numerical Function." In *Decision Process*, 159–65. New York: John Wiley.

Edwards, W. 1954. "The Theory of Decision Making." In *Decision Making*. Penguin Books.

Fishburn, Peter C. 1970. *Utility Theory for Decision Making*. Research Analysis Corporation.

Kreps, David M. 1988. *Notes on the Theory of Choice*. Underground Classics in Economics. Boulder: Westview Press.

———. 1990. *A Course in Microeconomic Theory*. London: Harvester Wheatsheaf.

Luce, R. Duncan, and Horward Raiffa. 1957. *Games and Decisions*. John Wiley and Sons, Inc.

Luce, R. Duncan, and Patrick Suppes. 1964. "Preference, Utility, and Subjective Probability." In *Handbook of Mathematical Psychology*. Volume III. John Wiley and Sons, Inc.

May, K. O. 1954. "Intransitivity, Utility, and the Aggregation of Preference Patterns." *Econometrica* 22.

Peters, Michael. 2005. "Honours Intermediate Microeconomics." http://montoya.econ.ubc.ca/book.php.

Quesada, Antonio. 2010. "On the Non-Nonequivalence of Weak and Strict Preference," March. http://gandalf.fee.urv.cat/professors/AntonioQuesada/171.1.pdf.

Rechenauer, Martin. 2008. "On the Non-Equivalence of Weak and Strict Preference." *Mathematical Social Sciences* 56 (3): 386–88. doi:10.1016/j.mathsocsci.2008.05.001.

Rigotti, Luce. 2014. "Preference and Utility." http://www.pitt.edu/~luca/ECON2100/lecture_03.pdf.

Rubinstein, Ariel. 2006. *Lecture Notes in Microeconomic Theory: The Economic Agent*. Princeton Paperbacks. Princeton, N.J: Princeton University Press.

Savage, L. J. 1954. *The Foundations of Statistics*. New York: John Wiley.

Savage, Richard P., Jr. 1994. “The Paradox of Nontransitive Dice.” *The American Mathematical Monthly* 101 (5): 429–36. doi:10.2307/2974903.

Tversky, Amos, and Daniel Kahneman. 1981. “The Framing of Decisions and the Psychology of Choice.” *Science* 211 (4481): 453–58. doi:10.1126/science.7455683.

———. 1986. “Rational Choice and the Framing of Decisions.” In *Choices, Values, and Frames*. Cambridge University Press.

Varian, Hal R. 2003. *Intermediate Microeconomics: A Modern Approach Sixth Edition*. W. W. Norton & Company, Inc.

Von Neumann, John, and Oskar Morgenstern. 1953. *Theory of Games and Economic Behavior*. Princeton University Press.

Weinstein, Arnold A. 1968. “Individual Preference Intransitivity.” *Southern Economic Journal* 34 (3): 335–43. doi:10.2307/1055496.